# Online Appendix
Firm Market Power, Worker Mobility, and Wages in the US Labor Market

# A Model Appendix

## A.1 Nash Bargaining

Claim: Suppose an employed worker at firm-$\theta_i$ has an outside option at firm-$\theta_j$. Then the Nash bargained wage, $\omega(\theta_i, \theta_j)$ solves equation 3.
Proof: Nash bargaining implies that the worker and firm negotiate a wage that solves the following objective function:

$$\max\left(W(\theta_i, \omega(\theta_i, \theta_j)) - \widetilde{W}(\theta_j, \omega(\theta_j, \theta_j), \theta_i)\right)^{\alpha}\left(J(\theta_i, \omega(\theta_i, \theta_j))\right)^{1-\alpha}$$
$$= \max\left[\alpha log\left(W(\theta_i, \omega(\theta_i, \theta_j)) - \widetilde{W}(\theta_j, \omega(\theta_j, \theta_j), \theta_i)\right) + (1-\alpha) log\left(J(\theta_i, \omega(\theta_i, \theta_j))\right)\right]$$

where $\omega(\theta_j, \theta_j) = \theta_j$. First order condition w.r.t. $\omega(\theta_i, \theta_j)$:

$$\alpha \frac{W_\omega(\theta_i, \omega(\theta_i, \theta_j))}{W(\theta_i, \omega(\theta_i, \theta_j)) - \widetilde{W}(\theta_j, \omega(\theta_j, \theta_j), \theta_i)} = -(1-\alpha)\frac{J_\omega(\theta_i, \omega(\theta_i, \theta_j))}{J(\theta_i, \omega(\theta_i, \theta_j))}$$

Note that $W_\omega(\theta_i, \omega(\theta_i, \theta_j)) = -J_\omega(\theta_i, \omega(\theta_i, \theta_j))$ from the expressions of $W$ and $J$ in equations 5 & 9.

$$\alpha J(\theta_i, \omega(\theta_i, \theta_j)) = (1-\alpha)\left(W(\theta_i, \omega(\theta_i, \theta_j)) - \widetilde{W}(\theta_j, \omega(\theta_j, \theta_j), \theta_i)\right)$$
$$W(\theta_i, \omega(\theta_i, \theta_j)) = \widetilde{W}(\theta_j, \omega(\theta_j, \theta_j), \theta_i) + \alpha\Big(W(\theta_i, \omega(\theta_i, \theta_j)) + J(\theta_i, \omega(\theta_i, \theta_j))$$
$$- \widetilde{W}(\theta_j, \omega(\theta_j, \theta_j), \theta_i)\Big)$$
$$W(\theta_i, \omega(\theta_i, \theta_j)) = \widetilde{W}(\theta_j, \omega(\theta_j, \theta_j), \theta_i) + \alpha\left(V(\theta_i) - \widetilde{W}(\theta_j, \omega(\theta_j, \theta_j), \theta_i)\right)$$

which simplifies to equation 3:

$$W(\theta_i, \theta_j) = \widetilde{W}(\theta_j, \theta_j, \theta_i) + \alpha\left(V(\theta_i) - \widetilde{W}(\theta_j, \theta_j, \theta_i)\right)$$

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## A.2 Solution Algorithm

The solution algorithm involves sequentially solving for $\theta_u$, and $\widetilde{V}$ through value function iteration. I write the following algorithm to solve the model numerically:

While $\widetilde{V}' \neq \widetilde{V}$ & $\theta'_u \neq \theta_u$ :

- Compute $\theta_u$ from equation 11.
- Update $\theta$, $n(\theta)$ and $f(\theta)$ grids and interpolate/extrapolate $\widetilde{V}$ to make it consistent with the updated grids. Denote the updated functions by $'$.
- Solve for $\widetilde{V}(\theta_j, \theta_i)$ for all $i \geq j$, as a function of $\widetilde{V}', \theta', n', f'$ from equation 10.
- Compute error and update: $\widetilde{V} = \widetilde{V}'$ and $\theta = \theta'$.

## A.3 Wage Function

In this section I derive the equilibrium wage function. For brevity, I denote $W(\theta_i, \theta_j) \equiv W_{ij}$, $\omega(\theta_i, \theta_j) \equiv \omega_{ij}$, $\widetilde{V}(\theta_j, \theta_i) \equiv V_{ji}$, $V(\theta_i) \equiv V_i$, and $f(\theta_i) \equiv f_i$.

Start with the worker value function in equation 5 and plugging in the Nash Bargaining equation:

$$\begin{aligned}(\gamma+\delta)W_{ij} = \omega_{ij} + \delta V_u + \lambda_1 \Bigg( &\sum_{x=i+1}^{N} \Big( (1-\alpha)V_{ix} + \alpha V_x - W_{ij} \Big) n_x f_x \\ &+ \sum_{x=j+1}^{i-1} \Big( (1-\alpha)V_{xi} + \alpha V_i - W_{ij} \Big) n_x f_x + \Big( V_i - W_{ij} \Big)(n_i - 1) f_i \Bigg)\end{aligned} \tag{A.1}$$

The value function of the worker can also be expressed as the following, combining equations 3 and 10:

$$\begin{aligned}(\gamma+\delta)W_{ij} &= (1-\alpha)\theta_j + \alpha\theta_i + \delta V_u \\ &+ (1-\alpha)\lambda_1 \Bigg( \sum_{x=j+1}^{N} \Big( (1-\alpha)V_{jx} + \alpha V_x - V_{ji} \Big) n_x f_x - \alpha(V_i - V_{ij}) f_i + (V_j - V_{ji})(n_j - 1) f_j \Bigg) \\ &+ \alpha\lambda_1 \sum_{x=i+1}^{N} \Big( (1-\alpha)V_{ix} + \alpha V_x - V_i \Big) n_x f_x\end{aligned} \tag{A.2}$$

Combining equations A.1 and A.2, the wage function can be expressed as:

$$
\begin{aligned}
\omega_{ij} =&(1-\alpha)\theta_j + \alpha\theta_i \\
&+ \lambda_1 \Bigg\{ (1-\alpha)\Bigg( \sum_{x=j+1}^{N} \Big( (1-\alpha)V_{jx} + \alpha V_x - V_{ji} \Big) n_x f_x - \alpha (V_i - V_{ji}) f_i + (V_j - V_{ji})(n_j - 1) f_j \Bigg) \\
&+ \alpha \sum_{x=i+1}^{N} \Big( (1-\alpha)V_{ix} + \alpha V_x - V_i \Big) n_x f_x - \sum_{x=i+1}^{N} \Big( (1-\alpha)V_{ix} + \alpha V_x - W_{ij} \Big) n_x f_x \\
&- \sum_{x=j+1}^{i-1} \Big( (1-\alpha)V_{xi} + \alpha V_i - W_{ij} \Big) n_x f_x - \Big( V_i - W_{ij} \Big)(n_i - 1) f_i \Bigg\}
\end{aligned}
\tag{A.3}
$$

Thus, the wage function, $\omega_{ij}$, $i \in \{\theta_u, \cdots, \theta_N\}, j \leq i$, can be expressed as a function of equilibrium outcomes $\widetilde{V}$ and $\theta_u$.

## A.4 Equilibrium Flows in the Labor Market

In this section, I describe the equilibrium flows of unemployed and employed workers in the labor market. Let $u$ denote the stock of the unemployed and let $e(\theta_i, \theta_j)$ denote the stock of workers employed at an individual firm with productivity $\theta_i$ and outside option $\theta_j$.

The size of such a firm is given by $E(\theta_i) = \sum_{j=u}^{i} e(\theta_i, \theta_j)$. Therefore, the total size of all firms at productivity level $\theta_i$ is $n_i E(\theta_i)$. Normalizing the total mass of workers to 1, the total employment can be expressed as $1 - u = \sum_{i=u+1}^{N} n_i E(\theta_i)$.

The equation for the law of motion of unemployment is:

$$\delta(1-u) = u\lambda_0 \sum_{x=u+1}^{N} n_x f(\theta_x) \tag{A.4}$$

On the left-hand side, it represents the inflows to unemployment, which is the stock of employed workers who separate from their jobs. On the right-hand side, it represents the outflows from unemployment, which is the stock of unemployed workers who find a job with a higher productivity level than their reservation threshold.

Next, consider the law of motion of employment. The stock of workers at $e(\theta_i, \theta_j)$ with $u <$

$j < i$, can be expressed as:

$$\lambda_1(1-\delta)\Bigg(\underbrace{n_j f(\theta_j)\sum_{x=u}^{j-1} e(\theta_i,\theta_x)}_{Stayers^+} + \underbrace{f(\theta_i) n_j E(\theta_j)}_{EE^+}\Bigg)$$
$$= e(\theta_i,\theta_j)\Bigg[\underbrace{\delta}_{EU} + (1-\delta)\lambda_1\Bigg(\underbrace{\sum_{x=i+1}^{N} n_x f(\theta_x)}_{EE^-} + \underbrace{\sum_{x=j+1}^{i-1} n_x f(\theta_x)}_{Stayers^-} + (n_i - 1) f(\theta_i)\Bigg)\Bigg] \tag{A.5}$$

The left-hand side of the equation includes two terms. The first term accounts for workers who stay in their current job and get promoted by receiving an offer from a more productive firm at productivity level $\theta_j$. The second term represents workers who switch jobs from any firm at productivity level $\theta_j$ to firm $\theta_i$.

On the right-hand side, the equation accounts for workers who leave their current position at $(i, j)$ either because they were separated to unemployed, or to move to a better firm, or get promoted to a better position within the same firm, or move to one of the peer firms at productivity level $\theta_i$. In the latter case, workers may choose to either stay or move to the new firm.

The stock of workers at $e(\theta_i, \theta_j)$ with $j = i$, can be expressed as:

$$\lambda_1(1-\delta)\Bigg(\underbrace{f(\theta_i)(n_i-1)\cdot(1-\nu)\cdot\sum_{x=u}^{i-1} e(\theta_i,\theta_x)}_{Stayers^+} + \underbrace{f(\theta_i)\cdot\nu\cdot(n_i-1)E(\theta_i)}_{EE^+}\Bigg)$$
$$= e(\theta_i,\theta_i)\Bigg[\underbrace{\delta}_{EU} + (1-\delta)\lambda_1\Bigg(\underbrace{\sum_{x=i+1}^{N} n_x f(\theta_x)}_{EE^-} + \underbrace{(n_i-1) f(\theta_i)\nu}_{EE^-}\Bigg)\Bigg] \tag{A.6}$$

The left-hand side's first term represents workers at the same firm-$\theta_i$, positioned below $i$, who receive an offer from any of the remaining firms at $i$ and stay with a promotion. The second term is composed of all workers at the remaining firms at $i$ who receive an offer from $i$ and decide to leave. The right-hand side encompasses all workers who leave for either a better firm or one of the peer firms at $i$.

Finally, the stock of workers at $e(\theta_i, \theta_j)$ with $j = u$, can be expressed as:

$$\underbrace{u\lambda_0 f(\theta_i)}_{UE} = e(\theta_i,\theta_u)\Bigg[\underbrace{\delta}_{EU} + (1-\delta)\lambda_1\Bigg(\underbrace{\sum_{x=i+1}^{N} n_x f(\theta_x)}_{EE^-} + \underbrace{\sum_{x=u+1}^{i-1} n_x f(\theta_x)}_{Stayers^-} + (n_i-1) f(\theta_i)\Bigg)\Bigg] \tag{A.7}$$

Combing the above equations, we can express law of motion of employment at firm-$\theta_i$:

$$\underbrace{u\lambda_0 f(\theta_i)}_{UE} + \lambda_1(1-\delta)\Bigg(\underbrace{f(\theta_i)\sum_{j=u+1}^{i-1} n_j E(\theta_j)}_{EE^+} + \underbrace{f(\theta_i)\cdot\nu\cdot(n_i-1)E(\theta_i)}_{EE^+}\Bigg)$$
$$= E(\theta_i)\Bigg[\underbrace{\delta}_{EU} + (1-\delta)\lambda_1\Bigg(\underbrace{\sum_{x=i+1}^{N} n_x f(\theta_x)}_{EE^-} + \underbrace{\nu(n_i-1)f(\theta_i)}_{EE^-}\Bigg)\Bigg] \tag{A.8}$$

Then we can define, EE transitions as:

$$EE^-(\theta_i) = E(\theta_i)\lambda_1(1-\delta)\Bigg(\underbrace{\sum_{x=i+1}^{N} n_x f(\theta_x)}_{\text{vertical EE}} + \underbrace{(n_i-1)f(\theta_i)\nu}_{\text{horizontal EE}}\Bigg) \tag{A.9}$$

$$EE^+(\theta_i) = \lambda_1(1-\delta)f(\theta_i)\Bigg(\underbrace{\sum_{j=u+1}^{i-1} n_j E(\theta_j)}_{\text{vertical EE}} + \underbrace{(n_i-1)E(\theta_i)\nu}_{\text{horizontal EE}}\Bigg) \tag{A.10}$$

## A.5 Model: Shutting off the Retaliation Channel

In this section I describe a version of the model outlined in Section 2, with the retaliation channel shut off. The wage setting equation 3 is replaced by the following:

$$W(\theta_i,\theta_j) = V(\theta_j) + \alpha\cdot\big(V(\theta_i) - V(\theta_j)\big) \tag{A.11}$$

The worker negotiating with firm-$\theta_i$ with an outside offer from firm-$\theta_j \leq \theta_i$ is offered a wage by firm-$\theta_i$ that ensures that the value to the worker is a linear combination of her outside option, i.e., the entire match value offered by firm-$\theta_j$ and $\alpha$ fraction of the increment in joint value that results from matching with firm-$\theta_i$. The outside option, $V(\theta_j)$, now contains the possibility of matching with firm-$\theta_i$ again. Thus, for any firm-$\theta_j$, we can write the joint value to the worker and firm as:

$$\begin{aligned}(\gamma+\delta)V(\theta_j) &= y(\theta_j) + \delta U \\ &+ \lambda_1\Bigg\{\sum_{x=j+1}^{N}\Big(W(\theta_x,\theta_j) - V(\theta_j))\Big)n_x f(\theta_x)\Bigg\}\end{aligned} \tag{A.12}$$

Equation A.12 replaces equation 6 in this model. In other words, the joint value of the worker and firm-$\theta_i$ is the flow value from the match output produced, and the option value of the worker searching on-the-job and matching with any firm more productive than $\theta_j$. As the latter set includes $\theta_i$, equations A.11 and A.12 show that firm-$\theta_i$ now competes with it's own offer that lies in the worker's outside option.

The value to the employed worker and firm remain the same, as outlined in equations 5 and 9. For the unemployed worker, the following wage setting equation replaces equation 4:

$$W(\theta_i, \theta_u) = U + \alpha\big(V(\theta_i) - U\big) \tag{A.13}$$

Equation A.13 states that an unemployed worker and firm-$\theta_i$ negotiate a wage that offers the worker a linear combination of their value from unemployment and the joint match value of firm-$\theta_i$. The value from unemployment contains the possibility of a future offer from $\theta_i$ and the is given by equation 7.

Thus, the model can be solved block recursively combining equations A.11, A.12, A.13, and 7.

# B Data Appendix

## B.1 Data

I use data from several sources to measure the effect of the number of firms per worker on the pace of worker mobility, average wages, and wage growth. First, I use publicly available tabulations from Business Dynamics Statistics (BDS). The BDS is part of the Longitudinal Business Database (LBD) of the US Census Bureau. It covers approximately 98 percent of non-farm private-sector employer businesses in the US starting 1978. It contains information on stocks of firms, establishments, and employees, as of March 12 of each year, disaggregated by location and industry. An establishment is identified by its physical location where a business is conducted, whereas a firm is an organization consisting of one or more establishments under common ownership or control. Employees consist of those working full- and part-time on a payroll.

Second, I link the BDS data with worker mobility and wage tabulations made publicly available from the Longitudinal Employer-Household Dynamics (LEHD) administrative data program. The LEHD is a matched employer-employee database of the US Census Bureau, and draws from data collected by state unemployment insurance programs. The data covers approximately 95% of all private sector employment, as well as employment in state and local governments. The public tabulations provide quarterly counts and rates of job-to-job transitions. Like the BDS, disaggregated data is available by region and industry. Still, unlike the BDS, all states did not enter the LEHD program simultaneously, with the earliest states' data available starting from 2000.

To combine data from the BDS and LEHD with measures of worker and firm demographics, I use local labor market statistics from the Quarterly Workforce Indicators (QWI). The QWI is also sourced from the LEHD program, and the earliest states entered the sample in 1990. QWI provides data on the composition of the workforce by age, education, firm age, and firm size and is disaggregated by locations and industries.

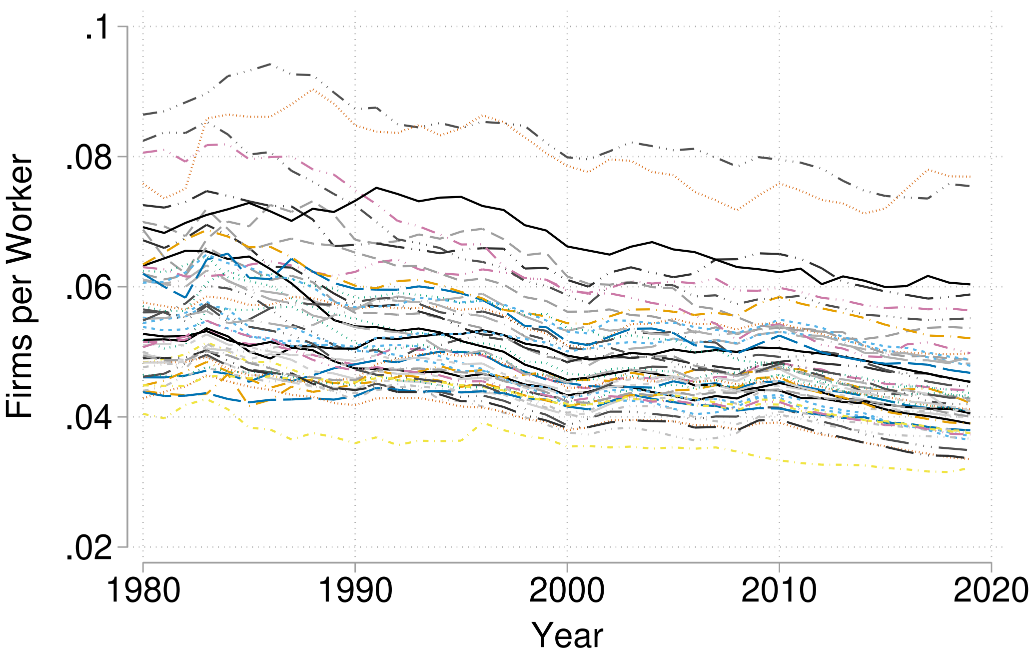
Combining the three data sources described above yields an annual panel of the sample period 2000-2018, with states entering the data at different times. The main variables of interest are measures of firms per worker, job-to-job flows, and employment composition by worker-age and education groups and firm-age and size groups. The combined dataset loses narrower levels of sectoral disaggregation that are available in some of the original sources. The most disaggregated data is available at the sector (two-digit NAICS industry) by MSA by year level. The overall dataset consists of an unbalanced panel of 381 MSAs, 18 industries over 19 years, yielding 124,750 sector-MSA-year observations.

To assess the model-implied behavior of wages relative to productivity, I combine data on firms per worker with the annual payroll share of gross value added. I use the data from the BLS at the disaggregated-industry level from 1987. The payroll share of value added is a measure of labor share published by the Bureau of Labor Statistics (BLS). Labor income is expressed as the sum of the compensation to employees on payroll and the compensation of the self-employed, and I focus on the former component.[1] The dataset contains a panel of about sixty industries.

To measure residual wage growth associated with job switches and job stays, I use microdata from the Survey of Income and Program Participation (SIPP) covering the period 1996-2000. The SIPP is a tri-annually collected, representative panel survey administered by the US Census Bureau, providing up 12 waves of individual data in the 1996 panel. Following Fujita and Moscarini (2017) I identify a primary job for each individual and define job spells and EE switches using job IDs and start and end dates of primary jobs. I merge the monthly SIPP data to firms per worker from the BDS at the state, sector, and year levels. For the main analysis, I consider the behavior of monthly wage growth for hourly workers and monthly earnings growth for non-hourly workers. Overall, the dataset contains about 50 thousand individual-1-year job spells and about 30 thousand instances of job-to-job transitions.

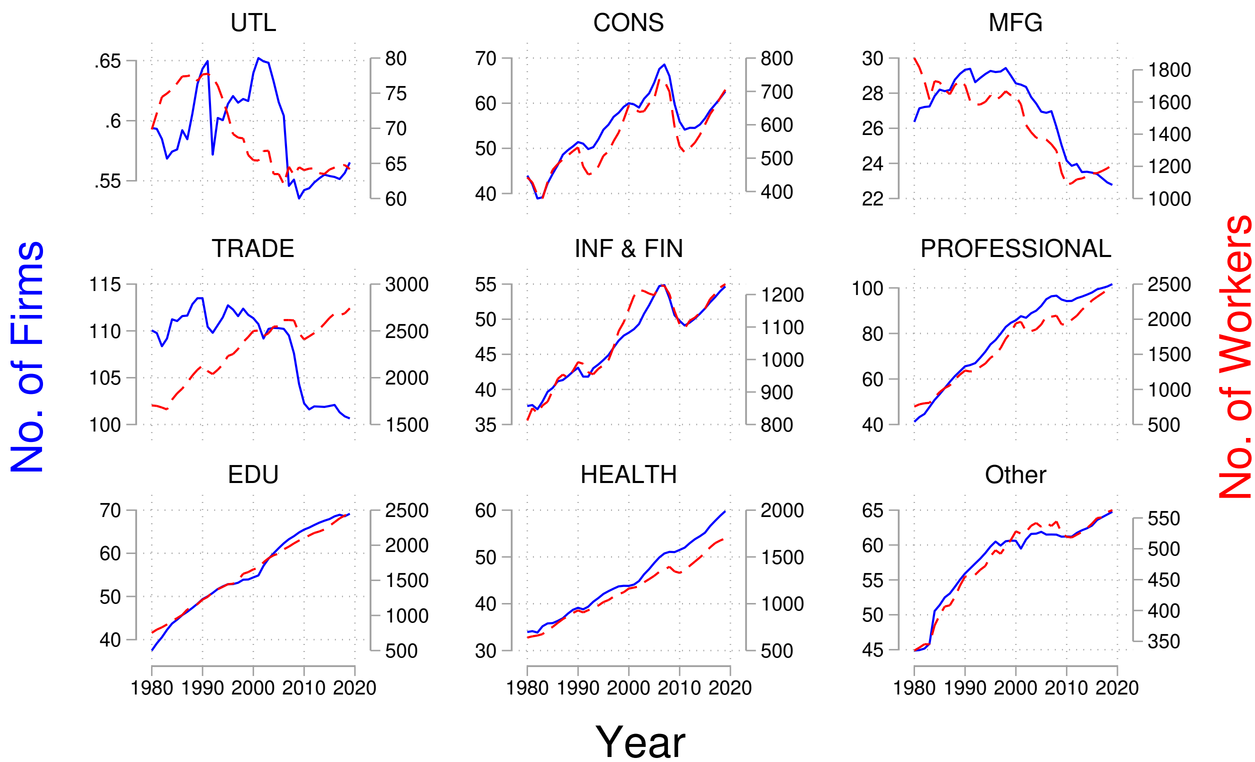
[1] Elsby, Hobijn, and Şahin (2013) provide a detailed account of each component of labor share, including its measurement and constituents.

Figure A1: Firms per Worker, state-wise, 1979-2018



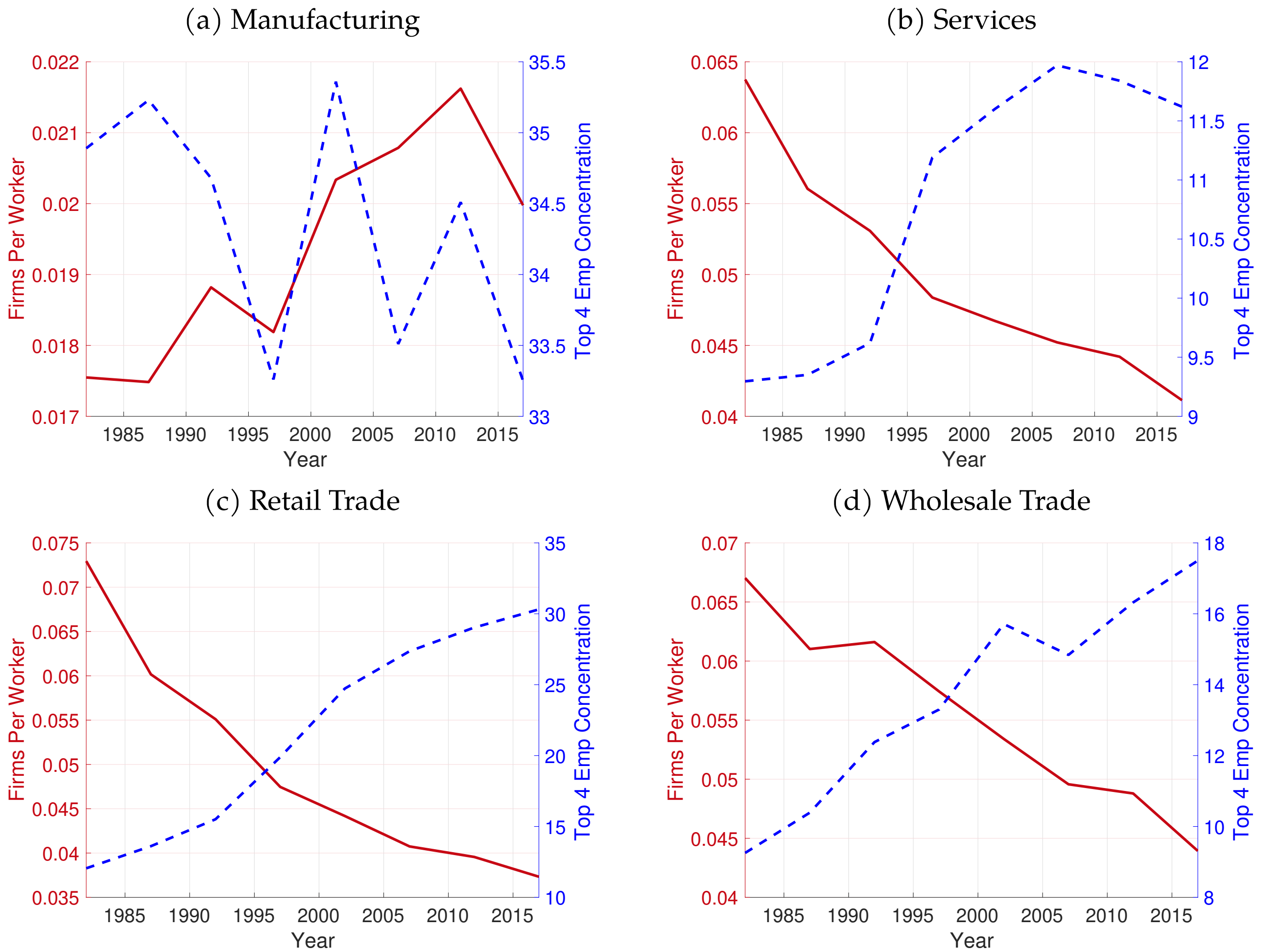

*Notes*: Business Dynamics Statistics. This figure shows the ratio of the number of firms to the number of workers for each state in the US economy from 1979-2018.

Figure A2: Number of Firms and Workers (in tens of thousands), 1979-2018



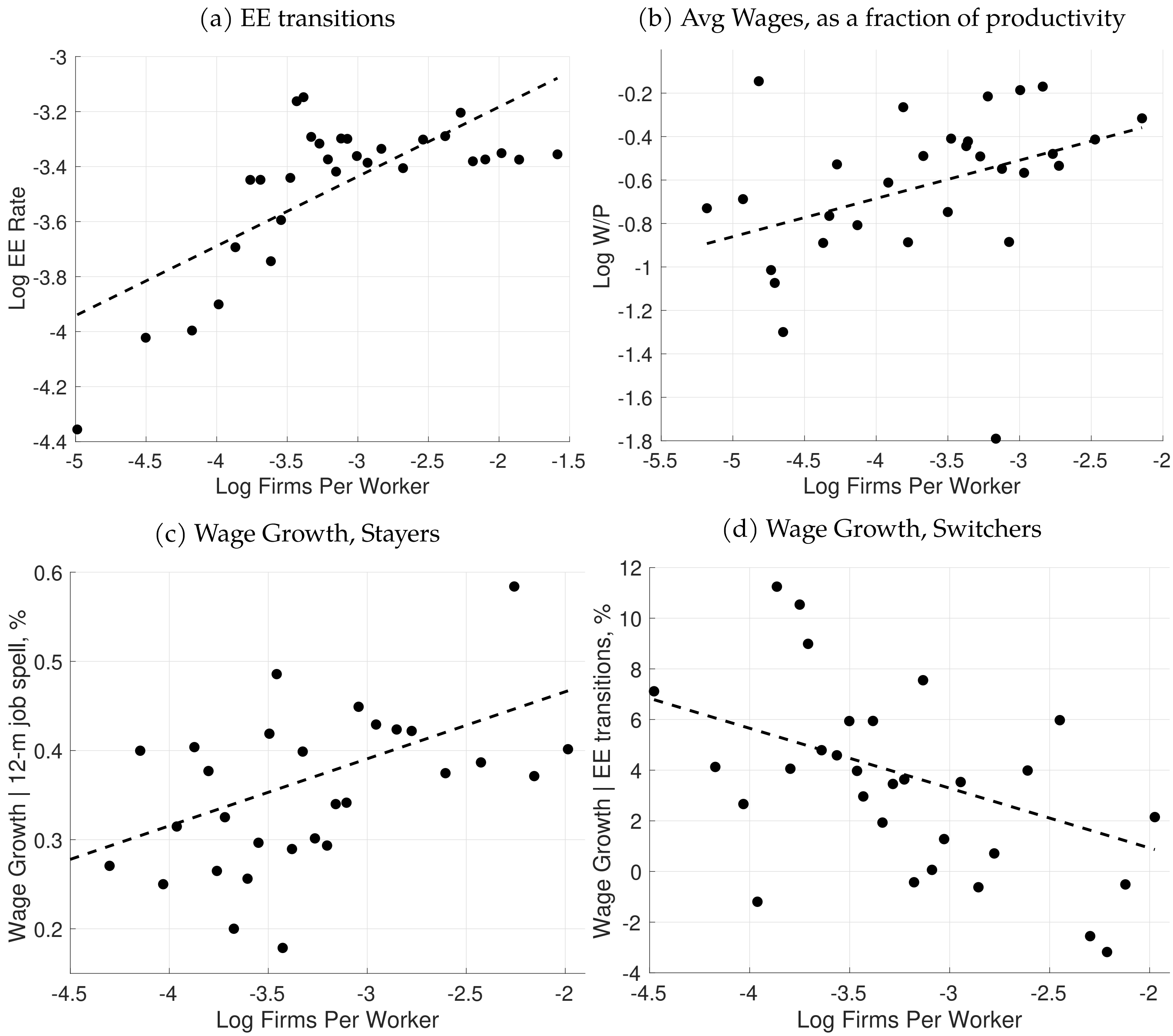

*Notes*: Business Dynamics Statistics. This figure plots the number of firms (left y-axis, in tens of thousands) and the number of workers (right y-axis, in tens of thousands) for each two-digit NAICS sector of the US economy over 1979-2018.

Figure A3: Employment share of the four largest firms from Autor et al. (2020) and Firms Per Worker




*Notes*: Business Dynamics Statistics. This figure displays the employment share accounted for by the four largest firms in each super-sector (Autor et al., 2020) and Firms per Worker within four-digit industries. Both indices are averaged across all four-digit industries to arrive at sector aggregates. The concentration index and firms per worker, respectively, weigh industries by their share of total sales and total employees.

Figure A4: Cross-sectional Correlations in the Data




*Notes*: This figure shows binned scatter plots the model-relevant outcome variables and firms per worker. Panel (a) plots the 2012-17 average of the firms per worker from the BDS and EE rates from the LEHD data across state × two-digit NAICS sector pairs. Panel (b) plots the 2012-17 average of the firms per worker and the payroll share of gross value added from the BLS across disaggregated industries. All variables are expressed in logs. Panel (c) and (d) present binned scatter plots of individual wage growth over a 12-month job spell and monthly wage growth associated with EE transitions from the SIPP against the firms per worker faced by the individual in their state and sector between 1996-2000.

# References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